## Human-Computer Interaction

# Statistics III <br> Intermediate Inferential Statistics 

 Professor Bilge Mutlu
## Today's Agenda

» Lecture: Multifactorial analysis + tutorials using example data
» Bonus Lecture: Writing and reporting
» Reminder: Please complete course evaluations at HelioCampus
(
COMP SCI 770-001 2024 Spring Ends: 5/3/2024 (9 days)


|  | Nominal | Categorical (2+) | Ordinal | Quantitative <br> Discrete | Quantitative Non- <br> Normal | Quantitative <br> Normal |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Nominal | Chi-squared, <br> Fisher's | Chi-squared | Chi-squared Trend, <br> Mann-Whitney | Mann-Whitney | Mann-Whitney, log- Student's <br> rank |  |
| Categorical (2+) | Chi-squared | Chi-squared | Kruskal-Wallis | Kruskal-Wallis | Kruskal-Wallis | ANOVA |
| Ordinal | Chi-squared Trend, <br> Mann-Whitney | Spearman rank | Spearman rank | Spearman rank | Spearman rank, |  |
| linear regression |  |  |  |  |  |  |

## Consider this dataset. Can we use multiple t-tests?

| Participant ID | Group | Time | Coding |
| :--- | :--- | ---: | ---: |
| Participant 01 | Standard | 245 | 0 |
| Participant 02 | Standard | 236 | 0 |
| Participant 03 | Standard | 321 | 0 |
| Participant 04 | Standard | 212 | 0 |
| Participant 05 | Standard | 267 | 0 |
| Participant 06 | Standard | 334 | 0 |
| Participant 07 | Standard | 287 | 0 |
| Participant 08 | Standard | 259 | 0 |
| Participant 09 | Prediction | 246 | 1 |
| Participant 10 | Prediction | 213 | 1 |
| Participant 11 | Prediction | 265 | 1 |
| Participant 12 | Prediction | 189 | 1 |
| Participant 13 | Prediction | 201 | 1 |
| Participant 14 | Prediction | 197 | 1 |
| Participant 15 | Prediction | 289 | 1 |
| Participant 16 | Prediction | 224 | 1 |
| Participant 17 | Speech-based dictation | 178 | 2 |
| Participant 18 | Speech-based dictation | 289 | 2 |
| Participant 19 | Speech-based dictation | 222 | 2 |
| Participant 20 | Speech-based dictation | 189 | 2 |
| Participant 21 | Speech-based dictation | 245 | 2 |
| Participant 22 | Speech-based dictation | 311 | 2 |
| Participant 23 | Speech-based dictation | 267 | 2 |
| Participant 24 | Speech-based dictation | 197 | 2 |
|  |  |  |  |

$H_{0}: \mu_{1}=\mu_{2}=\mu_{3}, \alpha=.05$

3 pairwise tests: $(1-\alpha)^{3}=0.86$
Reject $H_{0}$ when $p<0.14$ instead of $p<0.05$
$\rightarrow$ Type I error (reject $H_{0}$ when it is true)

| Participant ID | Group | Time | Coding |
| :---: | :---: | :---: | :---: |
| Participant 01 | Standard | 245 | 0 |
| Participant 02 | Standard | 236 | 0 |
| Participant 03 | Standard | 321 | 0 |
| Participant 04 | Standard | 212 | 0 |
| Participant 05 | Standard | 267 | 0 |
| Participant 06 | Standard | 334 | 0 |
| Participant 07 | Standard | 287 | 0 |
| Participant 08 | Standard | 259 | 0 |
| Participant 09 | Prediction | 246 | 1 |
| Participant 10 | Prediction | 213 | 1 |
| Participant 11 | Prediction | 265 | 1 |
| Participant 12 | Prediction | 189 | 1 |
| Participant 13 | Prediction | 201 | 1 |
| Participant 14 | Prediction | 197 | 1 |
| Participant 15 | Prediction | 289 | 1 |
| Participant 16 | Prediction | 224 | 1 |
| Participant 17 | Speech-based dictation | 178 | 2 |
| Participant 18 | Speech-based dictation | 289 | 2 |
| Participant 19 | Speech-based dictation | 222 | 2 |
| Participant 20 | Speech-based dictation | 189 | 2 |
| Participant 21 | Speech-based dictation | 245 | 2 |
| Participant 22 | Speech-based dictation | 311 | 2 |
| Participant 23 | Speech-based dictation | 267 | 2 |
| Participant 24 | Speech-based dictation | 197 | 2 |

What are errors in hypothesis testing?
Type I error: Rejecting $H_{0}$ when it is true
Type II error: Accepting $H_{0}$ when it is false

Type III error: Correctly rejecting $H_{0}$ for the wrong reason

|  | Null Hypothesis is true | Alternative Hypothesis is true |
| :--- | :--- | :--- |
| Fail to reject | Right decision | Wrong decision <br> Type II error <br> (False negative) |
| Reject | Wrong decision <br> Type I error <br> (False positive) | Right decision |

Analysis of Variance (ANOVA)
Definition: Analysis of variance (ANOVA) is a collection of statistical models and their associated estimation procedures (such as the "variation" among and between groups) used to analyze the differences among group means in a sample. ${ }^{1}$

## Procedures:

1. One-way (single factor)
2. Two-way (two factors)
3. Multi-way (multiple factors)

## Models:

1. Fixed effects (between)
2. Random effects (within)
3. Mixed effects (mixed)

How do we choose among these procedures?


## How do we conduct ANOVA?

We calculate the $F$-statistic.
$F=\frac{\sigma_{\text {explained }}}{\sigma_{\text {unexplained }}}=\frac{S S_{\text {treatment }} /(k-1)}{S S_{\text {error }} /(n-k)}$
$F=\frac{\sum n_{i}\left(M_{i}-\sum(M i / k)\right)^{2} /(k-1)}{\sum \sum\left(X_{i t}-M_{i}\right)^{2} /(n-k)}$

$k$ : number of populations
$n$ : sample size

## One-way ANOVA in R

```
model = aov(Time~Group,data=data)
summary(model)
    Df Sum Sq Mean Sq F value Pr(>F)
\begin{tabular}{lrrrrr} 
Group & 2 & 7842 & 3921 & 2.174 & 0.139
\end{tabular}
Residuals 21 37880 1804
```

| Participant ID | Group | Time | Coding |
| :--- | :--- | ---: | ---: |
| Participant 01 | Standard | 245 | 0 |
| Participant 02 | Standard | 236 | 0 |
| Participant 03 | Standard | 321 | 0 |
| Participant 04 | Standard | 212 | 0 |
| Participant 05 | Standard | 267 | 0 |
| Participant 06 | Standard | 334 | 0 |
| Participant 07 | Standard | 287 | 0 |
| Participant 08 | Standard | 259 | 0 |
| Participant 09 | Prediction | 246 | 1 |
| Participant 10 | Prediction | 213 | 1 |
| Participant 11 | Prediction | 265 | 1 |
| Participant 12 | Prediction | 189 | 1 |
| Participant 13 | Prediction | 201 | 1 |
| Participant 14 | Prediction | 197 | 1 |
| Participant 15 | Prediction | 289 | 1 |
| Participant 16 | Prediction | 224 | 1 |
| Participant 17 | Speech-based dictation | 178 | 2 |
| Participant 18 | Speech-based dictation | 289 | 2 |
| Participant 19 | Speech-based dictation | 222 | 2 |
| Participant 20 | Speech-based dictation | 189 | 2 |
| Participant 21 | Speech-based dictation | 245 | 2 |
| Participant 22 | Speech-based dictation | 311 | 2 |
| Participant 23 | Speech-based dictation | 267 | 2 |
| Participant 24 | Speech-based dictation | 197 | 2 |
|  |  |  |  |

One-way ANOVA in JMP

## Analyze > Fit X by Y

| Oneway Anova |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\checkmark$ Summary of Fit |  |  |  |  |  |  |  |
| Rsquare |  | 0.171518 |  |  |  |  |  |
| Adj Rsquare |  | 0.092615 |  |  |  |  |  |
| Root Mean Square Error |  |  | 42.47149 |  |  |  |  |
| Mean of Response |  |  | 245.125 |  |  |  |  |
| Observations (or Sum Wgts) |  |  | 24 |  |  |  |  |
| $\checkmark$ Analysis of Variance |  |  |  |  |  |  |  |
| Source | DF | Sum of Squares | Mean Square |  | F Ratio | Prob > F |  |
| Group | 2 | 7842.250 |  | 3921.13 | 2.1738 | 0.1387 |  |
| Error | 21 | 37880.375 |  | 1803.83 |  |  |  |
| C. Total |  | 45722.625 |  |  |  |  |  |
| - Means for Oneway Anova |  |  |  |  |  |  |  |
| Level |  | Number |  | Mean | Std Error | Lower 95\% | Upper 95\% |
| Prediction |  |  | 8 | 228.000 | 15.016 | 196.77 | 259.23 |
| Speech-based dicta |  |  | 8 | 237.250 | 15.016 | 206.02 | 268.48 |
|  |  |  | 8 | 270.125 | 15.016 | 238.90 | 301.35 |
| Std Error uses a pooled estimate of error variance |  |  |  |  |  |  |  |

Are we done?
The ANOVA analysis only told us whether the methods had a significant effect on time, not which method is more effective.

We can make two types of pairwise comparisons:

1. A priori comparisons (planned contrasts)

$$
H_{0}: \mu_{1}=\mu_{2} ; H_{1}: \mu_{1}>\mu_{2}
$$

2. Post hoc comparisons (exploratory pairwise tests)

Test $\mu_{1}$ VS $\mu_{2}, \mu_{1}$ VS $\mu_{3}, \mu_{2}$ VS $\mu_{3}$

## A priori comparisons in R

```
levels(data$Group)
comparison = c(1,-1,0)
mat = cbind(comparison)
contrasts(data$Group) <- mat
model = aov(Time~Group, data= data)
summary.aov(model, split = list(Group=list("mu1 vs mu2"=1)))
\begin{tabular}{lrrrrr} 
Group & 2 & 7842 & 3921 & 2.174 & 0.139 \\
\(\quad\) Group: mu1 vs mu2 & 1 & 342 & 342 & 0.190 & 0.668 \\
Residuals & 21 & 37880 & 1804 & &
\end{tabular}
```


## A priori comparisons in JMP

## Compare Means > Each pair, Student's t

## $\checkmark$ Means Comparisons

- Comparisons for each pair using Student's t
- Confidence Quantile
$2.07961 \quad 0.05$
$\checkmark$ LSD Threshold Matrix

| Abs(Dif)-LSD |  |  |  |
| :--- | :---: | ---: | ---: |
| Standard | -44.162 | -11.287 | -2.037 |
| Speech-based dictation | -11.287 | -44.162 | -34.912 |
| Prediction | -2.037 | -34.912 | -44.162 |

Positive values show pairs of means that are significantly different.

- Connecting Letters Report
Level Mean
Standard A 270.12500
Speech-based dictation A 237.25000
Speech-based dictation A 237.2500
Prediction A 228.00000
Levels not connected by same letter are significantly different.


## - Ordered Differences Report



## TukeyHSD(model)

## Tukey multiple comparisons of means <br> 95\% family-wise confidence level

Fit: aov(formula = Time ~ Group, data = data)
\$Group

|  | diff | lwr | upr | p adj |
| :--- | ---: | ---: | ---: | ---: |
| Speech-based dictation-Prediction | 9.250 | -44.27619 | 62.77619 | 0.9011856 |
| Standard-Prediction | 42.125 | -11.40119 | 95.65119 | 0.1409733 |
| Standard-Speech-based dictation | 32.875 | -20.65119 | 86.40119 | 0.2896872 |

## Post hoc comparison in JMP

## Compare Means > All Pairs, Tukey HSD

- Comparisons for all pairs using Tukey-Kramer HSD
- Confidence Quantile
$\mathbf{q}^{*}$ Alpha
$2.52057 \quad 0.05$
- HSD Threshold Matrix

Abs(Dif)-HSD

|  | Standard Speech-based dictation |  | Prediction |
| :--- | :---: | ---: | ---: |
| Standard | -53.526 | -20.651 | -11.401 |
| Speech-based dictation | -20.651 | -53.526 | -44.276 |
| Prediction | -11.401 | -44.276 | -53.526 |

Positive values show pairs of means that are significantly different.

- Connecting Letters Report Leve

Mean
Standard A 270.12500
Speech-based dictation A 237.25000
Prediction A 228.00000
Levels not connected by same letter are significantly different.

## - Ordered Differences Report



What if we had a within-participants design?
We conduct a repeated-measures or randomeffects one-way ANOVA.

| Participant ID | Group | Time | Coding |
| :---: | :---: | :---: | :---: |
| Participant 01 | Standard | 245 | 0 |
| Participant 01 | Prediction | 246 | 1 |
| Participant 01 | Speech-based dictation | 178 | 2 |
| Participant 02 | Standard | 236 | 0 |
| Participant 02 | Prediction | 213 | 1 |
| Participant 02 | Speech-based dictation | 289 | 2 |
| Participant 03 | Standard | 321 | 0 |
| Participant 03 | Prediction | 265 | 1 |
| Participant 03 | Speech-based dictation | 222 | 2 |
| Participant 04 | Standard | 212 | 0 |
| Participant 04 | Prediction | 189 | 1 |
| Participant 04 | Speech-based dictation | 189 | 2 |
| Participant 05 | Standard | 267 | 0 |
| Participant 05 | Prediction | 201 | 1 |
| Participant 05 | Speech-based dictation | 245 | 2 |
| Participant 06 | Standard | 334 | 0 |
| Participant 06 | Prediction | 197 | 1 |
| Participant 06 | Speech-based dictation | 311 | 2 |
| Participant 07 | Standard | 287 | 0 |
| Participant 07 | Prediction | 289 | 1 |
| Participant 07 | Speech-based dictation | 267 | 2 |
| Participant 08 | Standard | 259 | 0 |
| Participant 08 | Prediction | 224 | 1 |
| Participant 08 | Speech-based dictation | 197 | 2 |

```
model = aov(Time~Group+Error(Participant.ID/Group), data= data)
summary(model)
Error: Participant.ID
Df \begin{tabular}{l} 
Sum Sq Mean Sq F value \(\operatorname{Pr}(>F)\) \\
Residuals \\
7
\end{tabular} \(19113 \quad 2730\)
Error: Participant.ID:Group
    Df Sum Sq Mean Sq F value Pr(>F)
Group 2 7842 3921 2.925 0.0868 .
Residuals 14 18767 1341
Signif. codes: 0 ‘***’ 0.001 ‘**` 0.01 ‘*` 0.05 '.' 0.1 ‘' 1
```

Within-participants one-way ANOVA in JMP

## Using the Full Factorial Repeated Measures ANOVA Add-In:

Add-ins > Repeated Measures > Full-Factorial Design (Mixed Effects)

For additional options (e.g., comparisons):
Launch Dialog > Emphasis: Effect Leverage

| -Response Time |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - Effect Summary |  |  |  |  |  |  |  |
| $\checkmark$ Summary of Fit |  |  |  |  |  |  |  |
| RSquare 0. |  | 0.48879 |  |  |  |  |  |
| RSquare Adj 0.4 |  | 0.440103 |  |  |  |  |  |
| Root Mean Square Error 36 |  | 36.61292 |  |  |  |  |  |
| Mean of Response 2 |  | 245.125 |  |  |  |  |  |
| Observations (or Sum Wgts) 24 |  |  |  |  |  |  |  |
| - Parameter Estimates |  |  |  |  |  |  |  |
| - Random Effect Predictions |  |  |  |  |  |  |  |
| - REML Variance Component Estimates |  |  |  |  |  |  |  |
| Random Effect | Var Ratio | Var Component | Std Error | 95\% Lower | 95\% Upper | Wald p Value | Pct of Total |
| Participant ID 0.3456318Participant ID*Group |  | 463.32143 | 514.98022 | -546.0213 | 1472.6641 | 0.3683 | 25.685 |
|  |  | 1340.506 | 506.66363 | 718.52371 | 3334.1618 | <.0001* | 74.315 |
| Total -2 LogLikelihood $=224.2278050$ |  | 1803.8274 | 592.26174 | 1037.3604 | 3890.013 |  | 100.000 |
|  |  |  |  |  |  |  |  |
| Note: Total is the sum of the positive variance components. |  |  |  |  |  |  |  |
| Total including negative estimates $=1803.8274$ |  |  |  |  |  |  |  |
| - Covariance Matrix of Variance Component Estimates |  |  |  |  |  |  |  |
| Residual is confounded with Participant ID*Group and has been removed. |  |  |  |  |  |  |  |
| - Iterations |  |  |  |  |  |  |  |
| $\checkmark$ Fixed Effect Tests |  |  |  |  |  |  |  |
| Source Nparm DF | DF DFDen | F Ratio Prob | $b>$ |  |  |  |  |
| Group 2 | 214 | 2.92510. | 0868 |  |  |  |  |
| - Effect Details |  |  |  |  |  |  |  |

$\checkmark$ Response Time

- Effect Summary

Summary of Fit
0.4887

Root Mean Square Error $\quad 36.6129$
Mean of Response
. 45
Observations (or Sum Wgts)

- Random Effect Predictions
- REML Variance Component Estimates


## Between-participants two-way ANOVA in R

| Task type | Entry <br> method | Participant <br> Number | Task <br> time | Task Type <br> coding | Entry Method <br> coding |
| :--- | :--- | :--- | :---: | :---: | :---: |
| Transcription | Standard | Participant 1 | 245 | 0 | 0 |
| Transcription | Standard | Participant 2 | 236 | 0 | 0 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| Transcription | Prediction | Participant 9 | 246 | 0 | 1 |
| Transcription | Prediction | Participant 10 | 213 | 0 | 1 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |  |
| Transcription | Speech-based | Participant 17 | 178 | 0 | 2 |
| Transcription | dictation |  |  |  |  |
|  | Speech-based | Participant 18 | 289 | 0 | 2 |
| $\ldots$ | dictation | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| Composition | Standard | Participant 25 | 256 | 1 | 0 |
| Composition | Standard | Participant 26 | 269 | 1 | 0 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |  |
| Composition | Prediction | Participant 33 | 265 | 1 | 1 |
| Composition | Prediction | Participant 34 | 232 | 1 | 1 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |  |
| Composition | Speech-based | Participant 41 | 189 | 1 | 2 |
| Composition | dictation | Speech-based | Participant 42 | 321 | 1 |
| dictation | $\ldots$ | $\ldots$ | 2 |  |  |
| Composition | $\ldots$ | Speech-based | Participant 48 | 202 | 1 |

model $=\operatorname{aov}(T i m e \sim G r o u p * E x p e r t i s e, ~ d a t a=d a t a) ~$ summary(model)

|  | Df | Sum Sq | Mean Sq F | value | $\operatorname{Pr}(>F)$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Group | 2 | 7842 | 3921 | 2.175 | 0.143 |
| Expertise | 1 | 1395 | 1395 | 0.774 | 0.391 |
| Group: Expertise | 2 | 4030 | 2015 | 1.117 | 0.349 |
| Residuals | 18 | 32455 | 1803 |  |  |

## Between-participants two-way ANOVA in JMP

## Analyze > Fit Model




## Within-participants two-way ANOVA in R

model $=\operatorname{aov(Time~(Group*Task)+Error(Participant.ID/(Group*Task)),~data=~data)~}$
summary (model)

| Participant ID | Group | Task | Time |
| :--- | :--- | :--- | ---: |
| Participant 01 | Standard | Complex | 285 |
| Participant 01 | Prediction | Complex | 160 |
| Participant 01 | Speech-based dictation | Complex | 201 |
| Participant 01 | Standard | Simple | 272 |
| Participant 01 | Prediction | Simple | 191 |
| Participant 01 | Speech-based dictation | Simple | 161 |
| Participant 02 | Standard | Complex | 189 |
| Participant 02 | Prediction | Complex | 250 |
| Participant 02 | Speech-based dictation | Complex | 178 |
| Participant 02 | Standard | Simple | 247 |
| Participant 02 | Prediction | Simple | 288 |
| Participant 02 | Speech-based dictation | Simple | 180 |
| Participant 03 | Standard | Complex | 233 |
| Participant 03 | Prediction | Complex | 285 |
| Participant 03 | Speech-based dictation | Complex | 225 |
| Participant 03 | Standard | Simple | 200 |
| Participant 03 | Prediction | Simple | 202 |
| Participant 03 | Speech-based dictation | Simple | 162 |



## Within-participants two-way ANOVA in JMP

## Add-ins > Repeated Measures > Full-Factorial Design (Mixed Effects)



| > Summary of Fit |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RSquare |  |  | 0.397171 |  |  |  |  |  |  |  |
| RSquare Adj |  |  | 0.325405 |  |  |  |  |  |  |  |
| Root Mean Square Error |  |  | 37.84614 |  |  |  |  |  |  |  |
| Mean of Response |  |  | 216.625 |  |  |  |  |  |  |  |
| Observation | (or Sum | Wgts) |  |  |  |  |  |  |  |  |
| - Parameter Estimates |  |  |  |  |  |  |  |  |  |  |
| Term |  |  |  |  | Estimate | Std E | rror DFDen | $t$ Ratio Pr | b> $\mid$ \| |  |
| Intercept |  |  |  |  | 216.625 | 4.636 | 8889 | 46.72 < | 001* |  |
| Group[Prediction] |  |  |  |  | 1.6875 | 9.976 | 625514 | 0.17 |  |  |
| Group[Speech-based dictation] |  |  |  |  | -7.875 | 9.976 | 625514 | -0.79 |  |  |
|  |  |  |  |  | 2.6666667 | 7.725 | 769 | 0.350 |  |  |
| Group[Prediction] ${ }^{\text {Task[ }}$ [Complex] |  |  |  |  | -2.229167 | 7.725 | 14 | -0.29 0 |  |  |
| Group[Speech-based dictation]*Task[Complex] |  |  |  |  | 8.4583333 | 7.725 | 14 | 1.090 |  |  |
| - Random Effect Predictions |  |  |  |  |  |  |  |  |  |  |
| - REML Variance Component Estimates |  |  |  |  |  |  |  |  |  |  |
| Random Effect |  | Var Ratio |  | Var |  | Error | 95\% Lower | 95\% Upper | Wald pValue | Pct of Total |
| Participant ID |  |  | -0.324559 | Compone | $4.875 \quad 323.0$ | 7858 | -1098.097 | 168.34739 | 0.1502 | 0.000 |
| Participant ID*Group |  |  | 0.3338216 | 478.14286477.556555454 |  | 4376 | -553.4719 | 1509.7577 | 0.3637 | 20.022 |
| Participant ID*TaskParticipant ID*Group*Task |  |  |  |  |  |  | -583.6208 | 1538.7339 | 0.3778 | 19.998 |
|  |  |  |  | 1432.3304 <br> 2388.0298 <br> 188 |  | 6999 | 767.74244 | 3562.5512 | <.0001* | 59.980 |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Note: Total is the sum of the positive variance components. Total including negative estimates $=1923.1548$ |  |  |  |  |  |  |  |  |  |  |
| - Covariance Matrix of Variance Component Estimates |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Residual is confounded with Participant ID*Group*Task and has been removed. |  |  |  |  |  |  |  |  |  |  |
| - Iterations |  |  |  |  |  |  |  |  |  |  |
| - Fixed Effect Tests |  |  |  |  |  |  |  |  |  |  |
| Source | Nparm | DF | DFDen | F Ratio | Prob $>\mathrm{F}$ |  |  |  |  |  |
| Group <br> Task <br> Group*Task | 2 | 2 | 14 | 0.3455 | 0.7138 |  |  |  |  |  |
|  | 1 | 1 | 7 | 0.1191 | 0.7401 |  |  |  |  |  |
|  | 2 | 2 | 14 | 0.6441 | 0.5400 |  |  |  |  |  |

## Two-way mixed-effects ANOVA in $R$

model $=\operatorname{aov}($ Time~(Group*Task)+Error(Participant.ID/Group)+Task,data=data)
summary (model)

| Participant ID | Group | Task | Time |
| :--- | :--- | :--- | ---: |
| Participant 01 | Standard | Complex | 285 |
| Participant 01 | Prediction | Complex | 160 |
| Participant 01 | Speech-based dictation | Complex | 201 |
| Participant 02 | Standard | Simple | 272 |
| Participant 02 | Prediction | Simple | 191 |
| Participant 02 | Speech-based dictation | Simple | 161 |
| Participant 03 | Standard | Complex | 189 |
| Participant 03 | Prediction | Complex | 250 |
| Participant 03 | Speech-based dictation | Complex | 178 |
| Participant 04 | Standard | Simple | 247 |
| Participant 04 | Prediction | Simple | 288 |
| Participant 04 | Speech-based dictation | Simple | 180 |
| Participant 05 | Standard | Complex | 233 |
| Participant 05 | Prediction | Complex | 285 |
| Participant 05 | Speech-based dictation | Complex | 225 |
| Participant 06 | Standard | Simple | 200 |
| Participant 06 | Prediction | Simple | 202 |
| Participant 06 | Speech-based dictation | Simple | 162 |


| Error: Participant.ID |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Df | Sum Sq | Mean Sq | value | $\operatorname{Pr}(>F)$ |
| Task | 1 | 341 | 341.3 | 0.175 | 0.682 |
| Residuals | 14 | 27279 | 1948.5 |  |  |
| Error: Participant.ID:Group |  |  |  |  |  |
|  | Df | Sum Sq | Mean Sq | F value | $\operatorname{Pr}(>F)$ |
| Group | 2 | 1650 | 825.2 | 0.432 | 0.654 |
| Group:Task | 2 | 1845 | 922.5 | 0.483 | 0.622 |
| Residuals | 28 | 53493 | 1910.5 |  |  |

## Two-way mixed-effects ANOVA in JMP

## Add-ins > Repeated Measures > Full-Factorial Design (Mixed Effects)



| - Summary of Fit |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RSquare |  |  | 0.057814 |  |  |  |  |  |  |
| RSquare Adj |  |  | -0.05435 |  |  |  |  |  |  |
| Root Mean Square Error |  |  | 43.70896 |  |  |  |  |  |  |
| Mean of Response |  |  | 216.625 |  |  |  |  |  |  |
| Observations | (or Sum | Wgts) | 48 |  |  |  |  |  |  |
| - Parameter Estimates |  |  |  |  |  |  |  |  |  |
| Term |  |  |  |  | Estimate St | rror DFDen | $t$ Ratio Pros | > $\mid$ \| |  |
| Intercept |  |  |  |  | 216.625 | $1352 \quad 14$ | 34.00 <. |  |  |
| Task[Complex] |  |  |  |  | 2.66666676 .3 | 135214 | 0.420. |  |  |
| Group[Prediction] |  |  |  |  | 2.6666875 1.6875 | 2054 | 0.190 .8 |  |  |
| Group[Speech-based dictation] |  |  |  |  | -7.875 8.9 | $2054 \quad 28$ | -0.88 0.3 |  |  |
| Task[Complex]*Group[Prediction] |  |  |  |  | -2.229167 8.92 | 8.922054 | -0.25 0.8 |  |  |
| Task[Complex]*Group[Speech-based dictation] |  |  |  |  | 8.45833338 .922 | 2054 | 0.950. |  |  |
| - Random Effect Predictions |  |  |  |  |  |  |  |  |  |
| - REML Variance Component Estimates |  |  |  |  |  |  |  |  |  |
| Random Effect |  |  | Var Ratio | Compone | Var | 95\% Lower | 95\% Upper | Wald $p$ Value | Pct of Total |
| Participant ID[Task] |  |  | 0.0066379 | 12.6815 | 1548298.71885 | -572.7966 | 598.15973 | 0.9661 | 0.659 |
| Participant ID*Group[Task] |  |  |  | 1910.4732510 .59544 |  | 1203.1556 | 3494.4955 | <.0001* | 99.341 |
| Total |  |  |  | 1923.15 | 1548419.68502 | 1307.4704 | 3106.8671 |  | 100.000 |
|  |  |  |  | -2 LogLikelihood = 457.81133323 |  |  |  |  |  |
| Note: Total is the sum of the positive variance components. Total including negative estimates $=1923.1548$ |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
| - Covariance Matrix of Variance Component Estimates |  |  |  |  |  |  |  |  |  |
| Residual is confounded with Participant ID*Group[Task] and has been removed. |  |  |  |  |  |  |  |  |  |
| - Iterations |  |  |  |  |  |  |  |  |  |
| - Fixed Effect Tests |  |  |  |  |  |  |  |  |  |
| Source | Nparm | DF | DFDen | F Ratio | Prob $>$ F |  |  |  |  |
| Task | 1 | 1 | 14 | 0.1752 | 0.6819 |  |  |  |  |
| Group | 2 | 2 | 28 | 0.4319 | 0.6535 |  |  |  |  |
| Task*Group | 2 | 2 | 28 | 0.4829 | 0.6221 |  |  |  |  |

What if I would like to include a covariate?

| Participant ID | Group | Time | Years |
| :--- | :--- | ---: | ---: |
| Participant 01 | Standard | 245 | 12 |
| Participant 02 | Standard | 236 | 5 |
| Participant 03 | Standard | 321 | 6 |
| Participant 04 | Standard | 212 | 13 |
| Participant 05 | Standard | 267 | 19 |
| Participant 06 | Standard | 334 | 18 |
| Participant 07 | Standard | 287 | 18 |
| Participant 08 | Standard | 259 | 18 |
| Participant 09 | Prediction | 246 | 14 |
| Participant 10 | Prediction | 213 | 3 |
| Participant 11 | Prediction | 265 | 19 |
| Participant 12 | Prediction | 189 | 13 |
| Participant 13 | Prediction | 201 | 24 |
| Participant 14 | Prediction | 197 | 21 |
| Participant 15 | Prediction | 289 | 5 |
| Participant 16 | Prediction | 224 | 18 |
| Participant 17 | Speech-based dictation | 178 | 21 |
| Participant 18 | Speech-based dictation | 289 | 18 |
| Participant 19 | Speech-based dictation | 222 | 23 |
| Participant 20 | Speech-based dictation | 189 | 16 |
| Participant 21 | Speech-based dictation | 245 | 12 |
| Participant 22 | Speech-based dictation | 311 | 15 |
| Participant 23 | Speech-based dictation | 267 | 16 |
| Participant 24 | Speech-based dictation | 197 | 9 |
|  |  |  |  |

Consider the one-way between-subjects analysis and also measuring the years of experience the user had in the task to control for that factor.

We conduct what is called an analysis of covariance (ANCOVA).

```
model = aov(Time~Group+Years, data=data)
summary(model)
\begin{tabular}{lrrrrr} 
Group & 2 & 7842 & 3921 & 2.090 & 0.15 \\
Years & 1 & 350 & 350 & 0.187 & 0.67 \\
Residuals & 20 & 37530 & 1877 & &
\end{tabular}
```

Because Years has no effect, we would remove it from our model (called model simplification) and rerun our analysis as an ANOVA.

## One-way between-participants ANCOVA in JMP

## Analyze > Fit Model




## Data files used in Statistics I, II, and III

